Gauss-Jordan Elimination

Finite Math

17 March 2017

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where m, n, p are real numbers and $p \neq 0$.

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Definition (Reduced Form)

A matrix is in reduced form if

1. *Each row consisting entirely of zeros is below any row having at least one nonzero element.*
2. *The leftmost nonzero element in each row is* 1*.*
3. *All other elements in the column containing the leftmost* 1 *of a given row are zeros.*
4. *The leftmost* 1 *in any row is to the right of the leftmost* 1 *in the row above.*

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$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

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$$\left[\begin{array}{cccc|c} 1 & 4 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

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$$\left[\begin{array}{cccc|c} 1 & 4 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & 4 & 8 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

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Example

Why are the following matrices not in reduced form? Put them in reduced form:

(a)

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 3 & -6 \end{array}\right]$$

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$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 3 & -6 \end{array}\right]$$

(b)

$$\left[\begin{array}{ccc|c} 1 & 5 & 4 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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(c)

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

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(c)

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

(d)

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

Gauss-Jordan Elimination

Let's now actually use Gauss-Jordan elimination to solve a system

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Example

Solve the following system using Gauss-Jordan elimination:

$$\begin{array}{rcrcrcl} 3x & + & y & - & 2z & = & 2 \\ x & - & 2y & + & z & = & 3 \\ 2x & - & y & - & 3z & = & 3 \end{array}$$

Gauss-Jordan Elimination

Example

Solve by Gauss-Jordan elimination:

$$\begin{array}{rcrcrcr} 2x_1 & - & 4x_2 & - & x_3 & = & -8 \\ 4x_1 & - & 8x_2 & + & 3x_3 & = & 4 \\ -2x_1 & + & 4x_2 & + & x_3 & = & 11 \end{array}$$

Now You Try It!

Example

Solve by Gauss-Jordan elimination:

$$\begin{array}{rcrcrcr} 3x_1 & + & 5x_2 & - & x_3 & = & -7 \\ x_1 & + & x_2 & + & x_3 & = & -1 \\ 2x_1 & & & + & 11x_3 & = & 7 \end{array}$$

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Example

Solve by Gauss-Jordan elimination:

$$\begin{array}{rcrcrcr} 3x_1 & + & 5x_2 & - & x_3 & = & -7 \\ x_1 & + & x_2 & + & x_3 & = & -1 \\ 2x_1 & & & + & 11x_3 & = & 7 \end{array}$$

Solution

$x_1 = -2, x_2 = 0, x_3 = 1$

Now You Try It!

Example

Solve by Gauss-Jordan elimination:

$$\begin{array}{rcrcrcl} 3x_1 & - & 4x_2 & - & x_3 & = & 1 \\ 2x_1 & - & 3x_2 & + & x_3 & = & 1 \\ x_1 & - & 2x_2 & + & 3x_3 & = & 2 \end{array}$$

Now You Try It!

Example

Solve by Gauss-Jordan elimination:

$$\begin{aligned} 3x_1 - 4x_2 - x_3 &= 1 \\ 2x_1 - 3x_2 + x_3 &= 1 \\ x_1 - 2x_2 + 3x_3 &= 2 \end{aligned}$$

Solution

No solution.

Now You Try It!

Example

Solve by Gauss-Jordan elimination:

$$\begin{aligned} 3x_1 - 4x_2 - x_3 &= 0 \\ 2x_1 - 3x_2 + x_3 &= 1 \\ x_1 - 2x_2 + 3x_3 &= 2 \end{aligned}$$

Now You Try It!

Example

Solve by Gauss-Jordan elimination:

$$\begin{array}{rcrcrcl} 3x_1 & - & 4x_2 & - & x_3 & = & 0 \\ 2x_1 & - & 3x_2 & + & x_3 & = & 1 \\ x_1 & - & 2x_2 & + & 3x_3 & = & 2 \end{array}$$

Solution

$x_1 = 7t - 4, x_2 = 5t - 3, x_3 = t$